Math 2410 - Calculus III

Final Exam Review Worksheet
Fall 2013, Dr. Graham-Squire

Work out each problem. When you finish, find the answer listed on the back page and its corresponding letter. Fill in that letter for each space where you find the question number. Question number 0 is done as an example.

0. Find the derivative. $f(x) = 2.7x$ Answer: $f'(x) =$ 2.7

1. A projectile is fired from ground level at an angle of 20° with the horizontal. The projectile has a range of 95 feet. Find the minimum initial velocity.

   Answer: _____

2. Find the curvature K of $\mathbf{r}(t) = \langle 2t, 5\cos t, 5\sin t\rangle$.

   Answer: _____

3. Find the limit and discuss the continuity of the function: $\lim_{(x,y)\to(0,0)} \dfrac{5x^2y}{x^2+y^2}$.

   Answer: _____

4. A right circular cone is measured and the radius and height are found to be $r = 2in$ and $h = 5in$. The possible error in each measurement is $\frac{1}{8}$ inch. Use differentials to approximate the maximum possible error in the calculation of the volume ($V = \frac{1}{3}\pi r^2 h$).

   Answer: _____

5. A team of oceanographers is mapping topography to assist in the recovery of a crashed helicopter. They develop the model

$$H = 250 + 30x^2 + 50\sin\frac{\pi y}{2}, \qquad 0 \le x \le 2, 0 \le y \le 2$$

   where H is height in meters (above sea level), and x and y are distances in *kilometers*. Suppose the helicopter is located at the point $x = 1$ and $y = 0.5$. How steep is the slope if you travel from the helicopter in the direction of the point (2,2)?

   Answer: _____

6. The production function for a candy manufacturer is $f(x,y) = 4x + xy + 2y$, where x is the number of units of labor and y is the number of unit of capital. Assume that units of labor cost \$20 and units of capital cost \$4, and the total amount of money available for both labor and capital is \$2000. Write a constraint equation and then find the maximum production level for this manufacturer. The answer is the value of the cost function at that production level.

Answer: _____

7. Evaluate the iterated integral. Change order of integration or coordinates as needed.

$$\int_0^2 \int_{x^2}^{2x} (x^2 + 2y)\, dy\, dx$$

Answer: _____

8. Evaluate the iterated integral. Change order of integration or coordinates as needed.

$$\int_0^4 \int_0^{\sqrt{16-y^2}} (x^2 + y^2)\, dy\, dx$$

Answer: _____

9. Find the volume of the solid bounded by the graphs of $z = x+y$, $z = 0$, $y = 0$, $x = 3$, and $y = x$.

Answer: _____

10. Find the volume of the solid bounded by the graphs of $z = 0$ and $z = 4$, outside the cylinder $x^2+y^2 = 1$ and inside the hyperboloid $x^2+y^2-z^2 = 1$. You can use a computer to help visualize the region, but you should be able to do the integral by hand.

Answer: _____

11. Find the area of the surface $f(x) = 4 - x^2$ over the region given by the triangle bounded by the graphs of $y = x$, $y = -x$, and $y = 2$. You can use a computer to integrate the integral.

Answer: _____

12. Evaluate the integral $\displaystyle\int_0^5 \int_0^{\sqrt{25-x^2}} \int_0^{\sqrt{25-x^2-y^2}} \frac{1}{1+x^2+y^2+z^2}\, dz\, dy\, dx$. Hint: once you have set up the integral correctly, it may be helpful to rewrite the integrand as $1 - \dfrac{1}{\text{something}}$.

Answer: _____

13. Use the change of variables $x = \frac{1}{2}(u+v)$, $y = \frac{1}{2}(u-v)$ to evaluate the double integral $\displaystyle\iint_R \ln(x+y)\, dA$ where R is the square region with corners at (1,2), (2,1), (3,2) and (2,3).

Answer: _____

14. Evaluate the line integral $\int_C xyz\,dx$ where C is described by

$$\mathbf{r}(t) = t\mathbf{i} + (t+2)\mathbf{j} + (2t-1)\mathbf{k}, \qquad 0 \le t \le 1$$

Answer: _____

15. Evaluate $\int_C 2xyz\,dx + x^2z\,dy + x^2y\,dz$ where C is the curve created by joining the line segments from the origin to (1,0,0), then from (1,0,0) to (1,3,0), then from (1,3,0) to (1,3,2).

Answer: _____

16. Evaluate the line integral $\int_C x^2y\,dx + (x^3 - y^3)\,dy$ where C is the triangle with vertices (0,0), (2,0), and (1,1).

Answer: _____

17. Let $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and let S be the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$. Evaluate

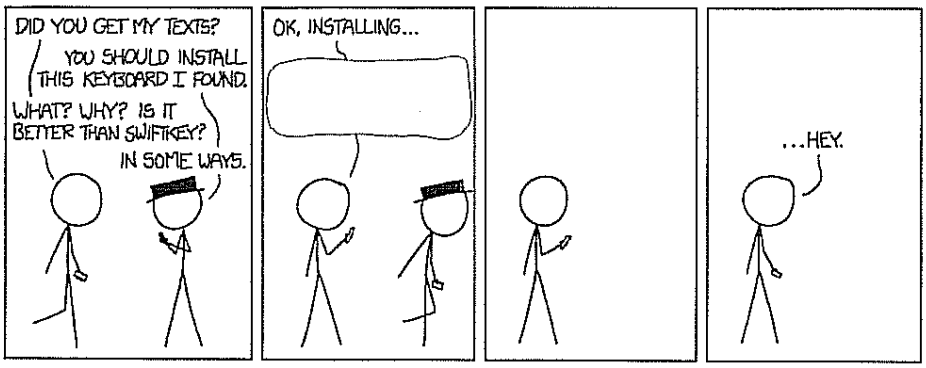
$$\iint_S \mathbf{F} \cdot \mathbf{N}\,dS$$

Use the divergence theorem to set up both integrals, and evaluate the one that you think is easiest.

Answer: _____

18. Let $\mathbf{F}(x,y,z) = (x-z)\mathbf{i} + (y-z)\mathbf{j} + x^2\mathbf{k}$ and S be the first octant portion of the plane $3x+y+2z = 12$. Use Stokes' theorem to set up integrals to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ as both a line integral and a double integral, then evaluate whichever you think is easier.

Answer: _____



xkcd.com/1284

Answer	Letter	Answer	Letter
2.7	I	100	A
6	B	2.751	Y
-6	A	3	C
128/3	S	1/25	O
-2/25	I	300	A
32π	K	600	D
27/2	I	$(120 + 37.5\pi\sqrt{2})\sqrt{13}$	T
8	P	700	T
-7	S	5/29	S
10	E	$(\pi/2)(5 - \tan^{-1} 5)$	E
$192\pi/5$	N	-4/45	T
$\pm\pi$	O	0	N
68.77	T	33/16	N
7.0717	H	$(4/3)(8\text{-}\sqrt{8})$	X
1/2	A	$64\pi/3$	G
$24320\pi/3$	E	$18\text{-}9\sqrt{2}$	R
88/15	R	$16\pi/15$	D
11/6	L	13,201.8	W

Fill in the blanks to figure out what the person is saying in the open bubble.

I

$\overline{0.5}$ $\overline{1}$ $\overline{2}$ $\overline{3}$ $\overline{4}$ $\overline{5}$ $\overline{6}$ $\overline{4}$ $\overline{7}$ $\overline{8}$ $\overline{9}$ $\overline{3}$ $\overline{10}$

$\overline{1}$ $\overline{11}$ $\overline{12}$ $\overline{8}$ $\overline{12}$ $\overline{13}$ $\overline{14}$ $\overline{7}$ $\overline{12}$ $\overline{14}$ $\overline{9}$ $\overline{2}$

$\overline{15}$ $\overline{16}$ $\overline{14}$ $\overline{3}$ $\overline{8}$ $\overline{9}$ $\overline{17}$ $\overline{14}$ $\overline{3}$ $\overline{5}$

$\overline{1}$ $\overline{13}$ $\overline{18}$ $\overline{12}$ $\overline{14}$ $\overline{3}$ $\overline{13}$ $\overline{5}$ $\overline{11}$ $\overline{9}$ $\overline{3}$ $\overline{10}$